UNIT-3

Expt. No.............................. Sheet No........................ Date ....

* Dynamic Programming :-

- Dynamic programming is a technique for solving problems with overlapping sub-problems. In this method, each sub-problem is solved only once. The result of each sub-problem
- DP is typically applied to optimizatn problems.
- DP uses bottom-up approach of problem solving (Iterative method).
- DP splits the i/p at every possible split points rather than at a particular point. After trying all the possibilities it determines which split point is ~~optai~~ optimal.

Difference between DAC and DP.

DAC (Divide & Conquer)	DP (Dynamic pgming)
1. Problem is divided into small sub-problems. These sub-problems solved independently all sub-problem of solⁿ are collected together to get the solⁿ.	1. DP have many decision sequence which are generated and all the overlapping solⁿˢ are to be considered.
2. DAC uses Top-down approach to solve a problem i.e., recursive method.	2. DP uses bottom-up approach to solve the problem. i.e., Itterative method.

* Elements of Dynamic Programming :-

1. Optimal Sub-Structure

The DP technique makes use of principle of optimality to find the optimal Sol^n from sub-problems.

2. Overlapping Sub-problems

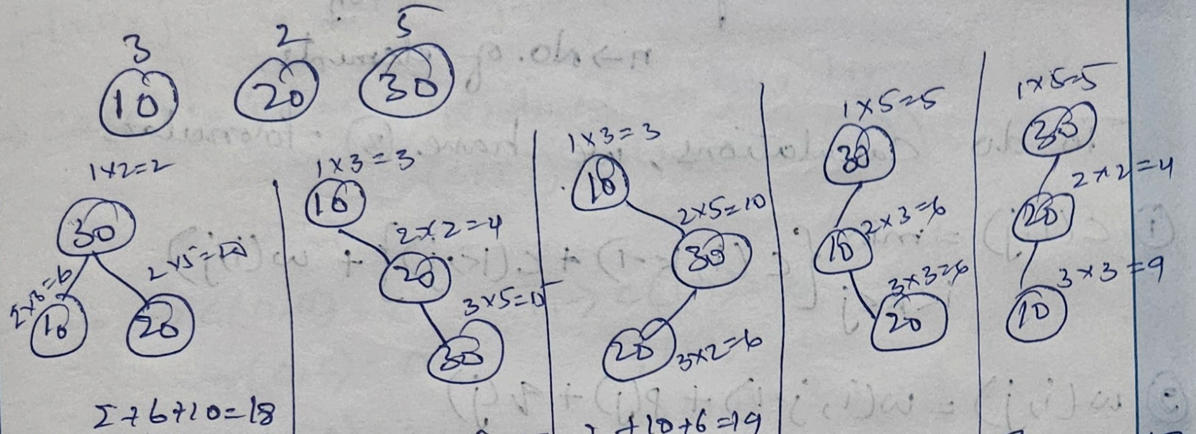
The DP is a technique in which the problem is divided into sub-problems. The Sol^{ns} of sub-problems are shared to get the final Sol^n to the problem. It avoids the repetition of work and we can get the Sol^n more efficiently.

* Applications

1. Optimal Binary Search trees.
2. 0/1 knapsack problem.
3. All pairs shortest path algorithm.
4. Travelling Salesperson problem.
5. Reliability design.

Expt. No. .... Sheet No. .... Date ....

Optimal Binary Search Tree



$2+6+10=18$

K	C
10	2
20	1
30	2
Avg	5/3 = 1.6

$3+4+15=22$

10	1
20	2
30	3
Avg	6/3 = 2

$3+10+6=19$

10	1
20	2
30	3
Avg	6/3 = 2

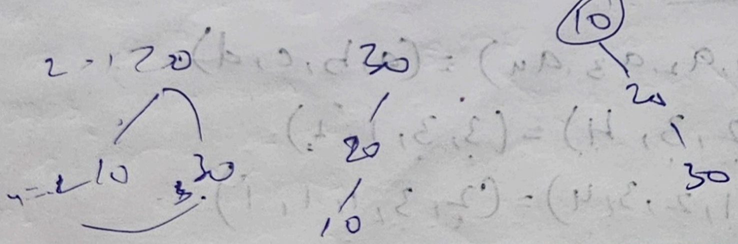
$5+6+6$
$=17$

Avg = 2

$5+4+9=18$

Avg = 2

10, 20, 30



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→ Optimal Binary Search Tree (OBST) using dynamic programming:

Each tree is represented as T_{0n}
$n \to$ no. of elements.

To do calculations, we have 3 formulae

1. $c(i,j) = \min\limits_{i<k\leq j} \{c(i,k-1) + c(k,j)\} + w(i,j)$

2. $w(i,j) = w(i,j-1) + p(j) + q(j)$
   p, q are probabilities; p = successful search
   q = Unsuccessful search.

3. $r(i,j) = k$

Binary search tree → elements less than root, left side
greater than root, right side.

optimal → For given set of numbers, we can build multiple binary trees.

But, among all → min no. of steps to reach the key.

Example problem:

$n=4$; $(a_1, a_2, a_3, a_4) = (a, b, c, d)$
$p(1,2,3,4) = (3,3,1,1)$
$q(0,1,2,3,4) = (2,3,1,1,1)$
Construct the OBST.

Expt. No............................ Sheet No...................... Date ....

$\boxed{n=4 \rightarrow T_{04} \rightarrow T_{0n}}$

$j-i=0,\ j-i=1,\ j-i=2,\ j-i=3,\ j-i=4$

→ case 1 :- $j-i=0$; $T_{00}, T_{11}, T_{22}, T_{33}, T_{44}$ (T_{ij})

$(c(i,j), w(i,j), r(i,j))$

$T_{00} \rightarrow c(0,0)=0$
$r(0,0)=0$
$w(0,0)=2$

$T_{11} \rightarrow c(1,1)=0$
$r(1,1)=0$
$w(1,1)=3$

Initially
$c(i,i)=0$
$r(i,i)=0$
$w(i,i)=q(i)$

$T_{22} \rightarrow c(2,2)=0$
$r(2,2)=0$
$w(2,2)=1$

$T_{33} \rightarrow c(3,3)=0$
$r(3,3)=0$
$w(3,3)=1$

$T_{44} \rightarrow c(4,4)=0$
$r(4,4)=0$
$w(4,4)=1$

Write all the values.

T_{00}	T_{11}	T_{22}	T_{33}	T_{44}
$c(0,0)=0$ $r(0,0)=0$ $w(0,0)=2$	$c(1,1)=0$ $r(1,1)=0$ $w(1,1)=3$	$c(2,2)=0$ $r(2,2)=0$ $w(2,2)=1$	$c(3,3)=0$ $r(3,3)=0$ $w(3,3)=1$	$c(4,4)=0$ $r(4,4)=0$ $w(4,4)=1$

Case 2 :- $j-i=1 \Rightarrow T_{01}, T_{12}, T_{23}, T_{34}$

For $T_{01} \rightarrow w(0,1) = w(0,0) + p(1) + q(1)$

$= 2 + 3 + 3 = 8$

$w(0,1) = 8$

$c(0,1) = \min_{0<k\leq 1} \{c(0,0) + c(1,1)\} + w(0,1)$

$k=1$

$0 + 0 + 8$

$c(0,1) = 8$

$r(i,j) = k = 1$

For $T_{12} \rightarrow w(1,2) = w(1,1) + p(2) + q(2)$

$= 3 + 3 + 1 = 7$

$w(1,2) = 7$

$c(1,2) = \min_{1<k\leq 2} \{c(1,1) + c(2,2)\} + w(1,2)$

$k=2$

$0 + 0 + 7$

$c(1,2) = 7$

$r(1,2) = k = 2$

For $T_{23} \rightarrow w(2,3) = w(2,2) + p(3) + q(3)$

$= 1 + 1 + 1 = 3$

$w(2,3) = 3$

$c(2,3) = \min_{2<k\leq 3} \{c(2,2) + c(3,3)\} + w(2,3)$

$k=3$

$0 + 0 + 3 = 3 \Rightarrow c(2,3) = 3$

$r(2,3) = k = 3$

For $T_{34} \rightarrow w(3,4) = w(3,3) + p(4) + q(4)$

$= 1 + 1 + 1 = 2$

$w(3,4) = 3$

$$c(3,4) = \min_{3<k\leq 4} \{c(3,3) + c(4,4)\} + w(3,4)$$

$k = 4$

$0 + 0 + 3$

$c(3,4) = 3$

$r(3,4) = k = 4$

$j-i=1$

T_{01}	T_{12}	T_{23}	T_{34}
$c(0,1) = 8$	$c(1,2) = 7$	$c(2,3) = 3$	$c(3,4) = 3$
$r(0,1) = 1$	$r(1,2) = 2$	$r(2,3) = 3$	$r(3,4) = 4$
$w(0,1) = 8$	$w(1,2) = 7$	$w(2,3) = 3$	$w(3,4) = 3$

Case 3 :- $j - i = 2 \Rightarrow T_{02}, T_{13}, T_{24}$

For $T_{02} \rightarrow w(0,2) = w(0,1) + P(2) + q(2)$

$= 8 + 3 + 1 = 12$

$$c(0,2) = \min_{0<k\leq 2} \{c(0,1) + c(2,2)\} + w(0,2)$$

$k = 2$

$8 + 0 + 12 = 20$

$$c(0,2) = \min_{\substack{0<k\leq 2 \\ k=1,2}} \begin{Bmatrix} k=1, & c(0,0)+c(1,2) \\ k=2, & c(0,1)+c(2,2) \end{Bmatrix} + w(0,2)$$

$$= \min \begin{Bmatrix} k=1, & 0+7=7 \\ k=2, & 8+0=8 \end{Bmatrix} + 12$$

min

$$= 7+12 = 19$$

$$c(0,2) = 19$$

$$r(0,2) = k = 1$$

For $T_{13} \Rightarrow w(1,3) = w(1,2) + p(3) + q(3)$

$$= 7+1+1 = 9$$

$$w(1,3) = 9$$

$$c(1,3) = \min_{\substack{1<k\leq 3 \\ k=2,3}} \begin{Bmatrix} k=2, & c(1,1)+c(3,3) \\ k=3, & c(1,2)+c(3,3) \end{Bmatrix} + w(1,3)$$

$$= \min \begin{Bmatrix} k=2, & 0+3=3 \\ k=3, & 7+0=7 \end{Bmatrix} + 9$$

min

$$c(1,3) = 3+9 = 12$$

$$r(1,3) = k = 2$$

Expt. No.............. Sheet No.............. Date ....

For $T_{24} \rightarrow w(2,4) = w(2,3) + P(4) + q(4)$

$= 3 + 1 + 1 = 5$

$w(2,4) = 5$

$$c(2,4) = \min_{\substack{2<k\leq 4 \\ k=3,4}} \left\{ \begin{array}{l} k=3, \ c(2,2)+c(3,4) \\ k=4, \ c(2,3)+c(4,4) \end{array} \right\} + w(2,4)$$

$$= \min \left\{ \begin{array}{ll} k=3, & 0+3=3 \checkmark \\ k=4, & 3+0=3 \end{array} \right\} + 5$$

$\Rightarrow c(2,4) = 3+5 = 8$

$r(i,j) = r(2,4) = k = 3.$

T_{02}	T_{13}	T_{24}
$c(0,2)=19$	$c(1,3)=12$	$c(2,4)=8$
$r(0,2)=1$	$r(1,3)=2$	$r(2,4)=3$
$w(0,2)=12$	$w(1,3)=9$	$w(2,4)=5$

case 4:- $j-i=3, \rightarrow T_{03} \quad T_{14}$

For $T_{03} \rightarrow w(0,3) = w(0,2) + P(3) + q(3)$

$= 12 + 1 + 1 = 14$

$w(0,3) = 14$

$$c(0,3) = \min_{\substack{0<k\le 3 \\ k=1,2,3}} \begin{Bmatrix} k=1, & c(0,0)+c(1,3) \\ k=2, & c(0,1)+c(2,3) \\ k=3, & c(0,2)+c(3,3) \end{Bmatrix} + w(0,3)$$

$$= \min \begin{Bmatrix} k=1, & 0+12 = 12 \\ k=2, & 8+3 = 11 \\ k=3, & 19+0 = 19 \end{Bmatrix} \overset{\min}{} + 14$$

$$11+14 = 25$$

$$c(0,3) = 25$$

$$r(0,3) = k = 2$$

For $T_{14} \rightarrow w(1,4) = w(1,3) + p(3) + q(3)$

$$= 9+1+1 = 11$$

$$w(1,4) = 11$$

$$c(1,4) = \min_{\substack{1<k\le 4 \\ k=2,3,4}} \begin{Bmatrix} k=2, & c(1,1)+c(2,4) \\ k=3, & c(1,2)+c(3,4) \\ k=4, & c(1,3)+c(4,4) \end{Bmatrix} + w(1,4)$$

$$= \min \begin{Bmatrix} k=2, & 0+8 = 8 \\ k=3, & 7+3 = 10 \\ k=4, & 12+0 = 12 \end{Bmatrix} \overset{\min}{} + 11$$

$$\Rightarrow c(1,4) = 8+11 = 19$$

$$r(1,4) = k = 2$$

$j-i=3$

T_{03}	T_{14}
$c(0,3)=25$	$c(1,4)=19$
$r(0,3)=2$	$r(1,4)=2$
$w(0,3)=14$	$w(1,4)=11$

case 5 :- $j-i=4 \to T_{04}$

For $T_{04} = w(0,4) = w(0,3)+p(4)+q(4)$

$$= 14+1+1=16$$

$$w(0,4)=16$$

$$c(0,4) = \min_{\substack{0<k\le 4 \\ k=1,2,3,4}} \left\{ \begin{array}{ll} k=1, & c(0,0)+c(1,4) \\ k=2, & c(0,1)+c(2,4) \\ k=3, & c(0,2)+c(3,4) \\ k=4, & c(0,3)+c(4,4) \end{array} \right\} + w(0,4)$$

$$= \min \left\{ \begin{array}{ll} k=1, & 0+19=19 \\ k=2, & 8+8=16 \checkmark \\ k=3, & 19+3=22 \\ k=4 & 25+0=25 \end{array} \right\} + 16$$

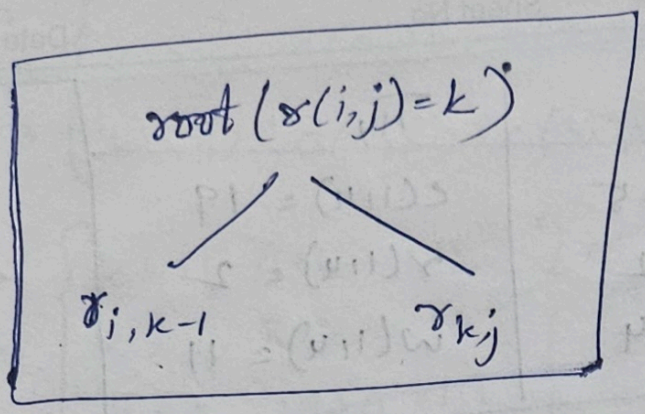
$$\Rightarrow c(0,4) = 16+16 = 32$$

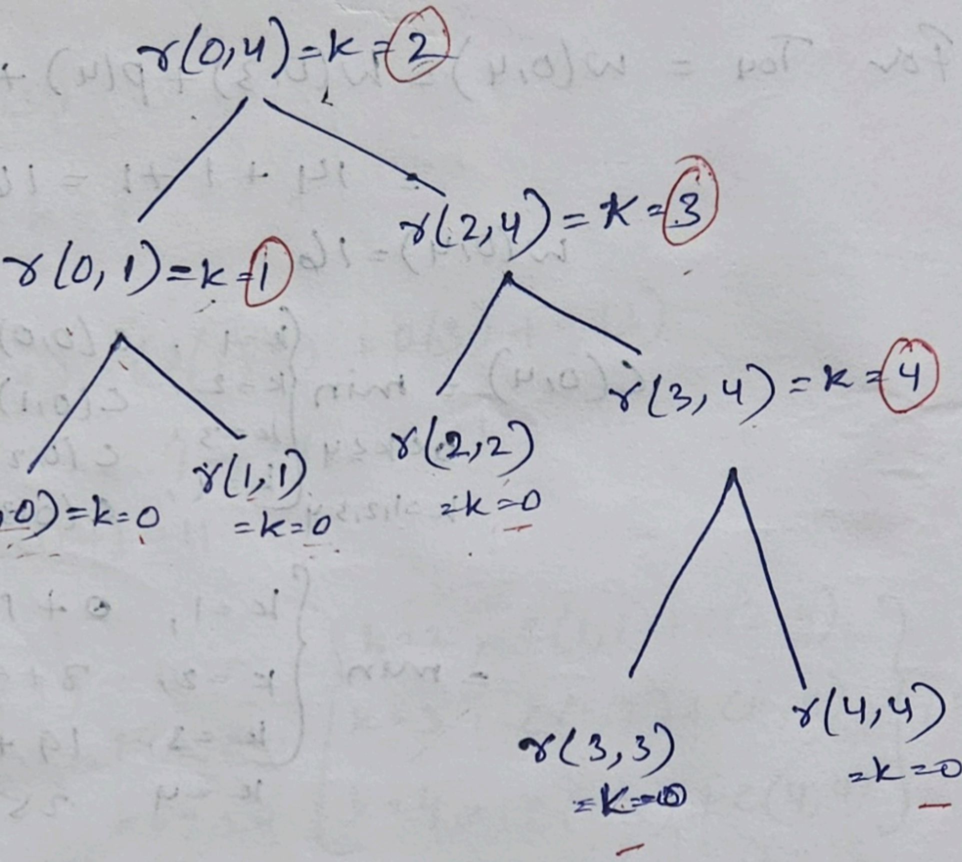
$$r(0,4)=k=2$$

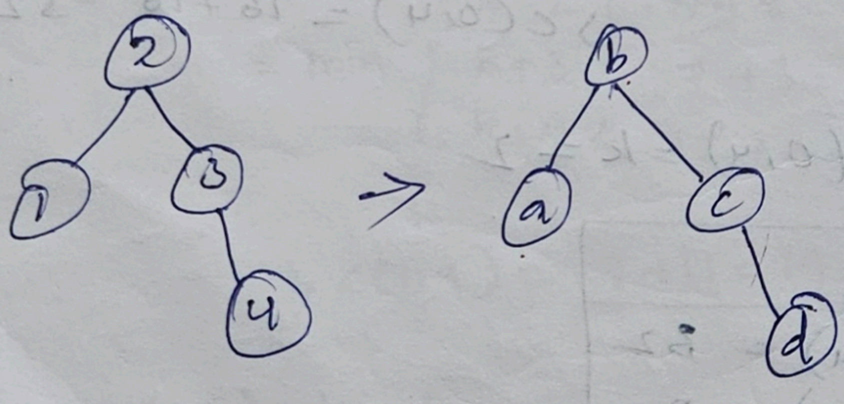
$j-i=4$

T_{04}
$c(0,4)=32$
$r(0,4)=2$
$w(0,4)=16$

General formula to construct OBST is:







Expt. No. .... Sheet No. .... Date ....

* 0/1 Knapsack Problem

If we are given 'n' objects and a knapsack or a bag in which the object i that has weight w_i is to be placed. The knapsack has a capacity W. Then the profit that can be earned is $P_i x_i$. The objective is to obtain filling of knapsack with maximum profit earned.

0/1 → either you pick the item completly or you don't pick them at all.

Eg:- Weights = {3, 4, 5, 6} and profit = {2, 3, 4, 1} respectively. Total weight = 8 and total items n = 4.

profit (P_i)	Weight (w_i)	weights →	0	1	2	3	4	5	6	7	8
		0	0	0	0	0	0	0	0	0	0
2	3	state 1	0	0	0	2	2	2	2	2	2
3	4	2	0	0	0	2	3	3	3	5	5
4	5	state 3	0	0	0	2	3	4	4	5	6
1	6	4	0	0	0	2	3	4	4	5	6

Max profit = 6

MRCE

$\max(3+0,2) = \max(3,2) = 3$

$\max(3+0,2) = 3$

$\max(3+0,2) = 3$

$\max(3+2,2) = 5$

$\max(3+2,2) = 5$

$\max(4+0,3) = 4$

$\max(4+0,3) = 4$

$\max(4+0,5) = 5$

$\max(4+2,5) = 6$

$\max(1+0,4) = 4$

$\max(1+0,5) = 5$

$\max(1+0,6) = 6.$

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x_1	x_2	x_3	x_4
1	0	1	0

$\{3, 4, 5, 6\}$

$\{2, 3, 4, 1\}$

$3+5=8$

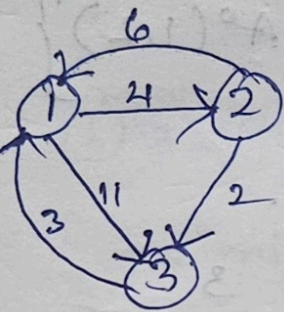
$6-4=2$

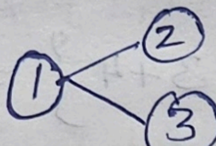
$2-2=0$

* All Pairs Shortest Path Problem

- We have to find the shortest path b/w every pair of vertices in a ~~direction~~ directed graph G.
- The all pair shortest path problem to determine a matrix A. such that A(i,j) is the length of shortest path i and j.

eg:-





Apply this formula
↓

$$A^k[i,j] = \min\left\{A^{k-1}[i,j],\ A^{k-1}[i,k] + A^{k-1}[k,j]\right\}$$

'k' is the intermediate vertex b/w i,j.

Write a matrix based on directed graph.

$$A^0 = \begin{array}{c|c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 4 & 11 \\ 6 & 0 & 2 \\ 3 & \infty & 0 \end{bmatrix} \end{array}$$

$\left\{\begin{array}{l} \text{No path} - \infty \\ \cdot\ \text{Diagonals} - 0 \end{array}\right\}$ Remember

$$A^1 = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 4 & 11 \\ 2 & 6 & 0 & \boxed{} \\ 3 & 3 & \boxed{} & 0 \end{array} \Rightarrow A^1 \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 4 & 11 \\ 2 & 6 & 0 & 2 \\ 3 & 3 & 7 & 0 \end{array}$$

$A^1(2,3) = \min\{A^0(2,3),\ A^0(2,1) + A^0(1,3)\}$

$= \min\{2, 6+11\} = \{2, 17\} \Rightarrow 2$.

$A^1(3,2) = \min\{A^0(3,2),\ A^0(3,1) + A^0(1,2)\}$

$= \min\{\infty, 3+4\} = 7$.

$$A^2 = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 4 & \boxed{} \\ 2 & 6 & 0 & 2 \\ 3 & \boxed{} & 7 & 0 \end{array} \Rightarrow \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 4 & 6 \\ 2 & 6 & 0 & 2 \\ 3 & 3 & 7 & 0 \end{array}$$

$A^2(1,3) = \min\{A^1(1,3),\ A^1(1,2) + A^1(2,3)\}$

$= \min\{11, 4+2\} = 6$

$A^2(3,1) = \min\{A^1(3,1),\ A^1(3,2) + A^1(2,1)\}$

$= \min\{3, 7+6\} = 3$

Expt. No. .... Sheet No. .... Date ....

$$A^3 = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & \square & 6 \\ \square & 0 & 2 \\ 3 & 7 & 0 \end{bmatrix} \end{array} \Rightarrow A^3 = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 4 & 6 \\ 5 & 0 & 2 \\ 3 & 7 & 0 \end{bmatrix} \end{array}$$

$$A^3(1,3) = \min\left\{A^2(1,2),\ A^2(1,3) + A^2(3,2)\right\}$$

$$= \min\{4, 6+7\} = 4$$

$$A^3(2,1) = \min\left\{A^2(2,1),\ A^2(2,3) + A^2(3,2)\right\}$$

$$= \min\{6, 2+3\} = 5$$

* Travelling Salesperson problem :- (TSP)

- Salesman will travel all the given cities and will come back to city he started.

eg:- (hyd, Blr, chennai, mumbai, delhi)

Eg: Robotics, network design, DNA sequencing

Started from hyd ←

Blr – chennai – mumbai – delhi

$\boxed{\text{Should choose path with minimum cost.}}$

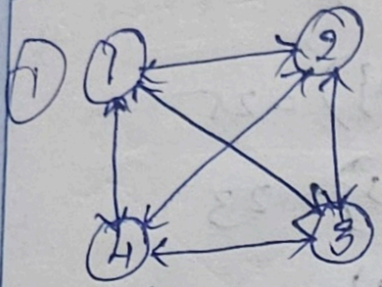
TSP is a NP-hard graph computational problem where the salesman must visit all cities (denoted using vertices in a graph) given in a set just once. The distances (denoted using edges in the graph) b/w all these cities are known. We are requested to find the shortest possible route in which the salesman visits all the cities and returns to the origin city.

In the following approach, we will solve the problem using the steps mentioned below:

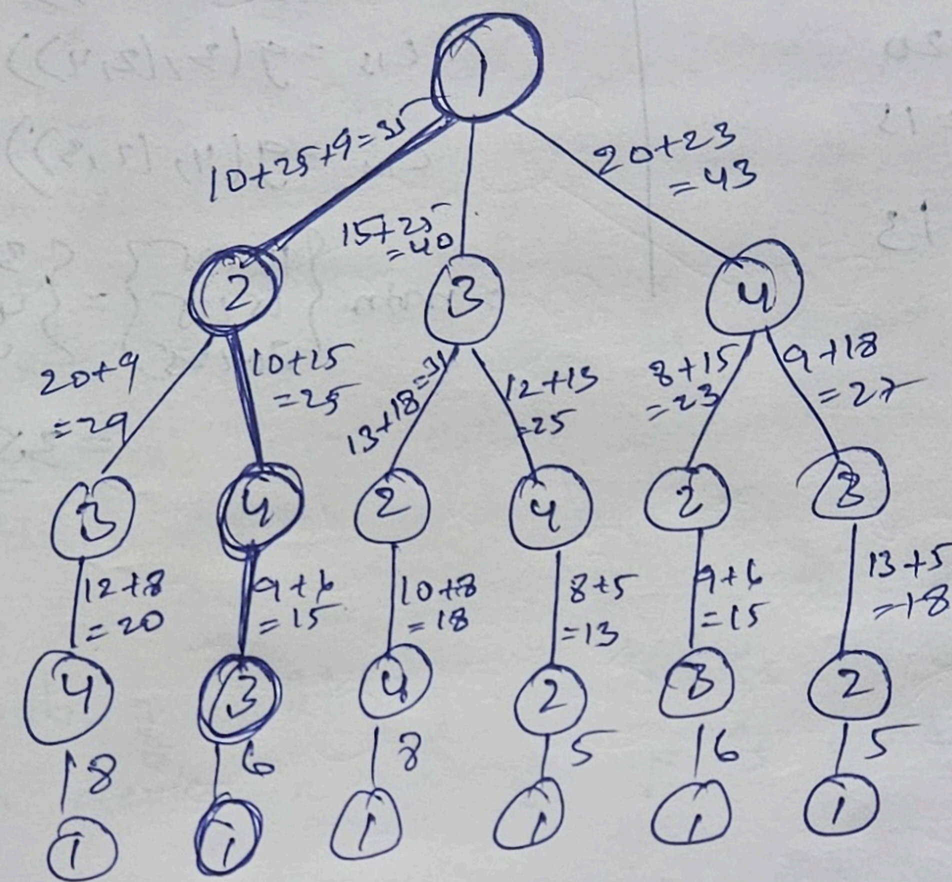
Steps:-

1. Firstly, we will consider city 1 as the starting and ending point. Since the route is cyclic, any point can be considered a starting point.
2. We will generate all the possible permutations of the cities, which are $(n-1)!$
3. After that, we will find the cost of each permutation and keep a record of the minimum cost permutation.
4. At last, we will return the permutations with minimum cost.

Expt. No.............. Sheet No.............. Date ....



$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 10 & 15 & 20 \\ 2 & 5 & 0 & 9 & 10 \\ 3 & 6 & 13 & 0 & 12 \\ 4 & 8 & 8 & 9 & 0 \end{array}$$



$$g(1,\{2,3,4\}) = \min_{k\in\{2,3,4\}} \{c_{ik} + g(k,\{2,3,4\}-\{k\})$$

$$g(i,s) = \min_{k\in s} \{c_{ik} + g(k, s-\{k\})$$

$g(2, \phi) = 5$

$g(3, \phi) = 6$

$g(4, \phi) = 8$

$g(2, \{3\}) = 15$

$g(2, \{4\}) = 18$

$g(3, \{2\}) = 18$

$g(3, \{4\}) = 20$

$g(4, \{3\}) = 15$

$g(4, \{2\}) = 13$

$g(2, \{3, 4\}) = 25$

$g(3, \{2, 4\}) = 25$

$g(4, \{2, 3\}) = 23$

$g(1, \{2, 3, 4\}) = \min($

$c_{12} + g(2, (3, 4))$

$c_{13} = g(3, (2, 4))$

$c_{14} = g(4, (2, 3))$

$= \min \begin{Bmatrix} 10+25 \\ 15+25 \\ 20+25 \end{Bmatrix} = \begin{Bmatrix} 35 \\ 40 \\ 45 \end{Bmatrix}$

$= \underline{\underline{35}}$

* Reliability Design

When devices are connected together then it is a necessity that each device should work properly. The probability that device 'i' will work properly is called reliability of that device.

Let r_i be the reliability of device D_i then reliability of entire system is πr_i. It may happen that even if reliability of individual device is very good but reliability of entire systems may not be good. Hence to obtain the good performance from entire system we can duplicate individual devices and can connect them in a series. To do so, we will attach switching circuits. The job of switching circuit is to determine which device in a group is working properly.

Setup a System

eg:- I'm Setting up a n/w
1. Computers
2. Internet
3. Routers
4. cables
} all should work fine

D_1	D_2	D_3	D_4
c_1	c_2	c_3	c_4
r_1	r_2	r_3	r_4
.9	.9	.9	.9

Product

$\rightarrow \prod r_i = 0.9^4 = 0.6561$

Suppose - $r_i = 0.9$

$1 - r_1 = 1 - 0.9 = 0.1$

$(1-r_1)^3 = (0.1)^3 = 0.001$

$1-(1-r_1)^3 = 0.999$

$\boxed{\text{1-Q in T/B}}$

Total available cost

$C = 105$ given

Q)

	D_i	C_i	r_i	u_i
1 ←	D_1	30	0.9	2
2 ←	D_2	15	0.8	3
2 ←	D_3	20	0.5	3

$$\Sigma c_i = c_1 + c_2 + c_3 = 30 + 15 + 20 = 65$$

$$C - \Sigma c_i = 105 - 65 = 40$$

floor values ←

$$u_i = \left\lfloor \frac{C - \Sigma c_i}{c_i} \right\rfloor + 1 = \frac{40}{30} = 1 + 1 = 2$$

$$\frac{40}{15} = 2 + 1 = 3$$

$$40/20 = 2 + 1 = 3$$

Initially
$(R, c) = (1, 0)$ min / max

for $D_1 \rightarrow$ 1 copy $\Rightarrow (0.9, 30)$

2 copies $\Rightarrow (0.99, 60)$

$R = 1-(1-r_1)^2$
$= 1-(1-0.9)^2$
$= 1-(0.1)^2$
$= 1-0.01$
$= 0.99$

order pair $\boxed{D_1 \Rightarrow (0.9, 30), (0.99, 60)}$

For $D_2 \rightarrow$ 1 copy $\Rightarrow (0.8 \times 0.9, 15+30), (0.8 \times 0.99, 15+60)$
(0.8, 15)
$(0.72, 45), (0.792, 75)$
1 2

2 copies $\Rightarrow (0.96 \times 0.9, 30+30),$
(0.96, 30)
$(0.96 \times 0.99, 30+60)$

$1-(1-0.8)^2 = 1-(0.2)^2$
$1-0.04 = 0.96$

$(0.864, 60), (0.9504, 90)$
3

$105 - 90 = 15$

3 copies $\Rightarrow (0.992 \times 0.9, 45+30),$
(0.992, 45)
$(0.992 \times 0.99, 45+60)$
105
$= (0.8928, 75)$
4

$1-(1-0.8)^3$
$= 1-(0.2)^3$
$= 1-0.008$
$= 0.992$

order pair $D_2 \rightarrow (0.72, 45)\boxed{(0.792, 75)}(0.864, 60)(0.8928, 75)$
use

$\boxed{D_2 \rightarrow (0.72, 45)(0.864, 60)(0.8928, 75)}$

For $D_3 \to$ 1 copy $\to (0.5 \times 0.72, 20+45), (0.5 \times 0.864, 20+60)$

$(0.5, 20)$ max ③

$(0.5 \times 0.8926, 20+75)$

$(0.36, 65)$ ① $(0.432, 80)$ ② $(0.4464, 95)$ ③

2 copies $\to$ $1-(1-r_1)^2$

$= 1-(1-0.5)^2$

$= 1-(0.5)^2$

$= 0.75$

2 copies $\to (0.75 \times 0.72, 40+45), (0.764 \times 0.75, 40+60)$

$(0.75, 40)$

$(0.54, 85)$ ④ $(0.648, 100)$ ⑤

3 copies $\to$ $1-(1-r_1)^3$

$1-(1-0.5)^3$

$1-(0.5)^3$

$1-0.125$

$= 0.875$

3 copies $(0.875, 60) \to (0.875 \times 0.72, 60+45)$, x, x

$= (0.63, 105)$ ⑥

$D_3 \to (0.36, 65) (0.432, 80) \boxed{(0.4464, 95)} (0.54, 85) \boxed{(0.63, 105)}$

$(0.648, 100)$

Max Reliability - 0.648

cost - 100

0.648 → D_3 - copies - 2.

0.864 → D_2 - copies - 2

0.9 → D_1 - copy - 1

0.54, 85 → D_3 - copies 2

0.72, 45 → D_2 - copy 1

0.9 → D_1 - copy 1